**ENGN2020 – HOMEWORK7**

### Problem 1

### Part (a)-Euler Method

### K21-1-1:

Answer:

It can be solved that , since , then .

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| step | xn | y'n | yn | exact values | error |
| 0 | 0 |  | 5 | 5.0000 | 0.0000 |
| 1 | 0.2 | -1 | 4.8 | 4.8039 | 0.0039 |
| 2 | 0.4 | -0.96 | 4.608 | 4.6156 | 0.0076 |
| 3 | 0.6 | -0.9216 | 4.4237 | 4.4346 | 0.0109 |
| 4 | 0.8 | -0.8847 | 4.2467 | 4.2607 | 0.0140 |
| 5 | 1.0 | -0.8493 | 4.0769 | 4.0937 | 0.0168 |
| 6 | 1.2 | -0.8154 | 3.9138 | 3.9331 | 0.0193 |
| 7 | 1.4 | -0.7828 | 3.7572 | 3.7789 | 0.0217 |
| 8 | 1.6 | -0.7514 | 3.6069 | 3.6307 | 0.0238 |
| 9 | 1.8 | -0.7214 | 3.4627 | 3.4884 | 0.0257 |
| 10 | 2.0 | -0.6925 | 3.3242 | 3.3516 | 0.0274 |

### K21-1-2:

Answer:

It can be solved that , since , then .

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| step | xn | y'n | yn | exact values | error |
| 0 | 0 |  | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.1 | 1.5708 | 0.1571 | 0.1564 | -0.0006 |
| 2 | 0.2 | 1.5513 | 0.3122 | 0.3090 | -0.0032 |
| 3 | 0.3 | 1.4923 | 0.4614 | 0.4540 | -0.0074 |
| 4 | 0.4 | 1.3936 | 0.6008 | 0.5878 | -0.0130 |
| 5 | 0.5 | 1.2557 | 0.7264 | 0.7071 | -0.0193 |
| 6 | 0.6 | 1.0796 | 0.8343 | 0.8090 | -0.0253 |
| 7 | 0.7 | 0.8659 | 0.9209 | 0.8910 | -0.0299 |
| 8 | 0.8 | 0.6122 | 0.9821 | 0.9511 | -0.0311 |
| 9 | 0.9 | 0.2955 | 1.0117 | 0.9877 | -0.0240 |
| 10 | 1 | 0.2409i | 1.0117+0.0241i | 1.0000 |  |

### Part (b)- Improved Euler Method

### K21-1-6:

Answer:

It can be solved that , since , then .

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| step | xn | yn | exact values | error |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0.05 | 0.1005 | 0.1003 | -0.0002 |
| 2 | 0.1 | 0.2030 | 0.2027 | -0.0003 |
| 3 | 0.15 | 0.3098 | 0.3093 | -0.0005 |
| 4 | 0.2 | 0.4234 | 0.4228 | -0.0006 |
| 5 | 0.25 | 0.5470 | 0.5463 | -0.0007 |
| 6 | 0.3 | 0.6849 | 0.6841 | -0.0008 |
| 7 | 0.35 | 0.8429 | 0.8423 | -0.0007 |
| 8 | 0.4 | 1.0299 | 1.0296 | -0.0002 |
| 9 | 0.45 | 1.2593 | 1.2602 | 0.0009 |
| 10 | 0.5 | 1.5538 | 1.5574 | 0.0036 |

### K21-1-7:

Answer:

It can be solved that , since , then .

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| step | xn | yn | exact values | error |
| 0 | 0 | 1.0000 | 1.0000 | 0.0000 |
| 1 | 0.1 | 1.0101 | 1.0050 | -0.0051 |
| 2 | 0.2 | 1.0309 | 1.0204 | -0.0105 |
| 3 | 0.3 | 1.0638 | 1.0471 | -0.0167 |
| 4 | 0.4 | 1.1110 | 1.0870 | -0.0241 |
| 5 | 0.5 | 1.1763 | 1.1429 | -0.0334 |
| 6 | 0.6 | 1.2654 | 1.2195 | -0.0459 |
| 7 | 0.7 | 1.3878 | 1.3245 | -0.0633 |
| 8 | 0.8 | 1.5600 | 1.4706 | -0.0894 |
| 9 | 0.9 | 1.8119 | 1.6807 | -0.1312 |
| 10 | 1 | 2.2050 | 2.0000 | -0.2050 |

### Part (c)- Comparison

### K21-1-9:

Answer: By using Euler Method, the result is shown as below:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| step | xn | yn | exact values | error |
| 0 | 0 | 1.0000 | 1.0000 | 0.0000 |
| 1 | 0.1 | 1.0100 | 1.0050 | -0.0050 |
| 2 | 0.2 | 1.0304 | 1.0204 | -0.0100 |
| 3 | 0.3 | 1.0623 | 1.0471 | -0.0151 |
| 4 | 0.4 | 1.1074 | 1.0870 | -0.0204 |
| 5 | 0.5 | 1.1687 | 1.1429 | -0.0258 |
| 6 | 0.6 | 1.2507 | 1.2195 | -0.0311 |
| 7 | 0.7 | 1.3601 | 1.3245 | -0.0356 |
| 8 | 0.8 | 1.5081 | 1.4706 | -0.0376 |
| 9 | 0.9 | 1.7129 | 1.6807 | -0.0322 |
| 10 | 1 | 2.0062 | 2.0000 | -0.0062 |

### K21-1-10:

Answer: By using Improved Euler Method with 20 steps, the result is shown as below:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| step | xn | yn | exact values | error |
| 0 | 0 | 1.0000 | 1.0000 | 0.0000 |
| 1 | 0.05 | 1.0025 | 1.0013 | -0.0013 |
| 2 | 0.1 | 1.0076 | 1.0050 | -0.0025 |
| 3 | 0.15 | 1.0152 | 1.0114 | -0.0039 |
| 4 | 0.2 | 1.0256 | 1.0204 | -0.0052 |
| 5 | 0.25 | 1.0390 | 1.0323 | -0.0067 |
| 6 | 0.3 | 1.0554 | 1.0471 | -0.0083 |
| 7 | 0.35 | 1.0753 | 1.0652 | -0.0100 |
| 8 | 0.4 | 1.0989 | 1.0870 | -0.0119 |
| 9 | 0.45 | 1.1267 | 1.1127 | -0.0141 |
| 10 | 0.5 | 1.1594 | 1.1429 | -0.0165 |
| 11 | 0.55 | 1.1975 | 1.1782 | -0.0193 |
| 12 | 0.6 | 1.2421 | 1.2195 | -0.0226 |
| 13 | 0.65 | 1.2944 | 1.2678 | -0.0265 |
| 14 | 0.7 | 1.3557 | 1.3245 | -0.0312 |
| 15 | 0.75 | 1.4282 | 1.3913 | -0.0369 |
| 16 | 0.8 | 1.5146 | 1.4706 | -0.0440 |
| 17 | 0.85 | 1.6186 | 1.5656 | -0.0530 |
| 18 | 0.9 | 1.7454 | 1.6807 | -0.0647 |
| 19 | 0.95 | 1.9026 | 1.8223 | -0.0802 |
| 20 | 1 | 2.1016 | 2.0000 | -0.1016 |